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# Geopositional Statistical Methods


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# Outline



- Background
- Sources of error in geopositional assessment
- Error model
- Discussion of geopositional error computation methods
- Modeled performance of geopositional error computation methods
- Conclusions and recommendations

# Background



- 1947 – U.S. Bureau of the Budget. National Map Accuracy Standards.
  - Establishes equivalent of circular error criteria as error standard of maps of various scales.
- 1962 – Clyde Greenwalt and Melvin Shultz. Principles of Error Theory and Cartographic Applications.
  - Provides rigorous treatment of circular error assuming that error is
    - Zero mean (no horizontal bias)
    - Normally distributed
    - Near-circular
- 1963 – Melvin Shultz. Circular Error Probability of a Quantity Affected by a Bias.
  - Provides limited treatment of error with horizontal bias.
- 1990 – MIL-STD-600001. Mapping, Charting and Geodesy Accuracy.
  - Adopts the 1963 Shultz approach to horizontal bias. Discusses empirical approach as an alternative estimate.
- 1998 – Federal Geographic Data Committee. National Standard for Spatial Data Accuracy (NSSDA).
  - Adopts Greenwalt and Shultz approach, but swaps RMSE for standard deviation. No provision for horizontal bias.
- 2003 – Joseph McCollum (USFS). Map Error and Root Mean Square.
  - Paper calls Greenwalt and Shultz method into question.
- 2003 – USGS Proposal for Revision of NSSDA.
  - Out of Geography Discipline. POC: John Conroy, jconroy@usgs.gov.
- 2004 (first version 2002?) – Tom Ager (NIMA InnoVision). An Analysis of Metric Accuracy Definitions and Methods of Computation.
  - White paper supports empirical approach. Also modifies Shultz approach to provide for large horizontal bias.

# Revision Status



- The revision of the NSSDA standard is currently in step 4, or the draft stage, of the 12-step FGDC standards approval process (http://www.fgdc.gov/standards/directives/dir1.html).
- Progress on the standard development will continue based on funding priorities.

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|---|
| **Proposal Stage** |
| Step 1, Develop Proposal |
| Step 2, Review Proposal |
| **Project Stage** |
| Step 3, Set Up Project |
| **Draft Stage** |
| Step 4, Produce Working Draft |
| Step 5, Review Working Draft |
| **Review Stage** |
| Step 6, Review and Evaluate Committee Draft |
| Step 7, Approve Standard for Public Review |
| Step 8, Coordinate Public Review |
| Step 9, Respond to Public Comments |
| Step 10, Evaluate Responsiveness to Public Comments |
| Step 11, Approve Standard for Endorsement |
| **Final Stage** |
| Step 12, Endorsement |

# Sources of Error in Geopositional Assessment



- Assessment Error
  - Ground Control Error
    - Pointing
    - Measurement
  - Analyst Error
    - Pointing
- Product Error (potential)
  - Spatial Resolution
  - Pointing (Displacement)
  - Azimuth
  - Scale
  - Orthogonality
  - Other product distortion
  - Terrain effects

- "Pointing error" for surveyors & analysts is here intended to mean the errors these individuals have in picking their target.

- **random error**

- "Measurement error" for ground control is here intended to mean the error inherent in the measuring instrument or system (GPS in this case).

- **constant systematic error**

- "Pointing error" for a geo-imaging system is here intended to mean the constant separation between estimated target coordinates and actual target coordinates.

- **functional systematic error**

# Check Point Error

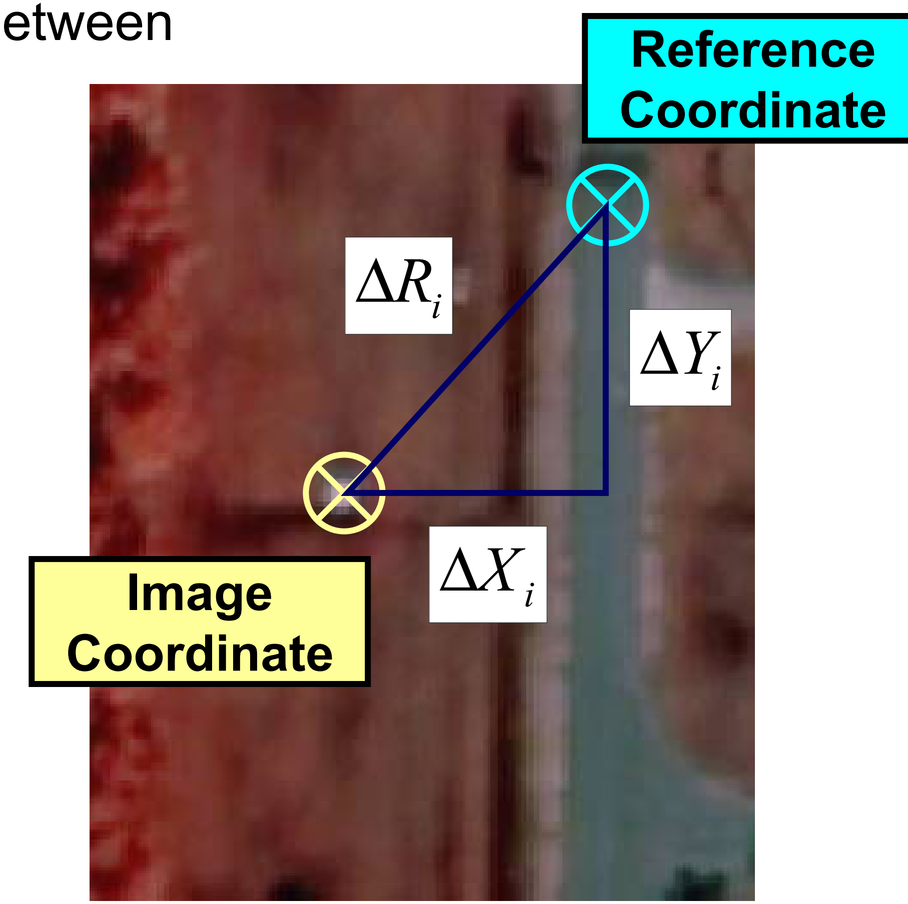
- Check Point Error – differences between image and reference coordinates

$$\Delta X_i = X_{image,i} - X_{reference,i}$$

$$\Delta Y_i = Y_{image,i} - Y_{reference,i}$$

- Check point error radial magnitude calculated by

$$\Delta R_i = \sqrt{\Delta X_i^{\,2} + \Delta Y_i^{\,2}}$$



# Error Component Estimates



- The error model chosen for generalized assessment

$$X_{image} = X + \varepsilon \quad \text{where} \quad \varepsilon = \varepsilon_{constant} + \varepsilon_{zero-mean}$$

- Horizontal Bias – an estimate of the constant error, designated here as $\mu_{H,}$ is the magnitude of the vector sum of the average error in the *X* and the *Y*

$$\mu_H = \sqrt{\left(\overline{\Delta X}\right)^2 + \left(\overline{\Delta Y}\right)^2}$$

- Circular Standard Error – an estimate of the zero-mean circular equivalent error valid even for elliptical error distributions with minimum to maximum error ratios as low as 0.6

$$\sigma_C \cong \frac{\sigma_{\Delta X} + \sigma_{\Delta Y}}{2} \quad \text{where} \quad \sigma_{\Delta X} = \sqrt{\frac{\sum\left(\Delta X_i - \overline{\Delta X}\right)^2}{n-1}} \quad \& \quad \sigma_{\Delta Y} = \sqrt{\frac{\sum\left(\Delta Y_i - \overline{\Delta Y}\right)^2}{n-1}}$$

Tom Ager used the horizontal error defined on the right, but Greenwalt and Shultz found this to be invalid for minimum to maximum error ratios less than 0.8.

$$\sigma_H = \sqrt{\frac{\left(\sigma_{\Delta X}^{\;2} + \sigma_{\Delta Y}^{\;2}\right)}{2}}$$

# RMSE Definitions



- RMSE – Root mean square error (horizontal bias & zero-mean error not decoupled)
  - 1D

$$RMSE_x = \sqrt{\sum \frac{\Delta X_i^2}{n}} \qquad RMSE_y = \sqrt{\sum \frac{\Delta Y_i^2}{n}}$$

  - 2D (NSSDA General)

$$RMSE_r = \sqrt{RMSE_x^2 + RMSE_y^2}$$

  - 2D (NSSDA Case 2*)

$$RMSE_c = 0.5 * (RMSE_x + RMSE_y)$$

* $RMSE_c$ is a recasting of terms in formula from NSSDA Appendix A Case 2. It is not found explicitly in the NSSDA.

# Circular Error Definitions



- $CE_{90}$ – The radial error which 90% of all errors in a circular distribution will not exceed (adapted from Greenwalt and Shultz, 1962)
  - Equivalent to the Circular Map Accuracy Standard (CMAS)
- $CE_{95}$ – The radial error which 95% of all errors in a circular distribution will not exceed (adapted from Greenwalt and Shultz, 1962)
  - Equivalent to $\text{Accuracy}_r$ (from NSSDA)
- In the normal case, circular error may be generally defined as the circle radius, $R$, that satisfies the conditions of the equation below (where *C.L.* is the desired confidence level ); however, *there is no analytical solution to this equation*.

$$C.L. = \int_{-R}^{R} \int_{\sqrt{R^2 - x^2}}^{\sqrt{R^2 - x^2}} \frac{1}{2\pi\sigma_x\sigma_y(1-\rho^2)} \exp\left[\frac{-1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right]\right] dydx$$

# Common $CE_{90}$ Estimates



- RMSE based (NSSDA)
  - Appendix A: General → $CE_{90} = 1.5175 \cdot \mathrm{RMSE}_r$
  - Appendix A: Case 2 → $CE_{90} = 2.1460 \cdot \mathrm{RMSE}_c$
- Bias and Standard Circular Error based
  - Sum of squares → $CE_{90} = \sqrt{(2.1460 \cdot \sigma_C)^2 + \mu_H{}^2}$
  - Shultz approach accounting for bias → $CE_{90} = 2.1272\sigma_C + 0.1674\mu_H + 0.3623\dfrac{\mu_H{}^2}{\sigma_C} - 0.055\dfrac{\mu_H{}^3}{\sigma_C{}^2}$
  - Ager approach accounting for bias (modified Shultz) →
    - When $\mu_H/\sigma_C \le 0.1$ $\quad CE_{90} = 2.1460\sigma_C$
    - When $0.1 < \mu_H/\sigma_C \le 3$ $\quad$ apply equation from Shultz
    - When $\mu_H/\sigma_C > 3$ $\quad CE_{90} = 0.986\mu_H + 1.4548\sigma_C$
- Empirically estimated
  - 90th percentile → $CE_{90} = 90^{th}$ percentile of $\Delta R$
  - Radial error for 1st point of percentile rank > 90

# Circular Error Modeling Study



- Assumed bivariate normal distribution of errors
- Modeled population (*all possible check points*) as 1M points
- Modeled sample (*simulated target range*) as 40 points (generated 10,000 trials of 40)
- Constrained $\sigma_C$ to 1 (*unitless for modeling purposes, but for spaceborne commercial imaging $\sigma_C$ ~ 1 meter*)
- Varied $\sigma_{min}/\sigma_{max}$ from 0 to 1 (*distributions from univariate through elliptical to perfectly circular*)
- Varied $\mu_H$ from 0 to 10,000

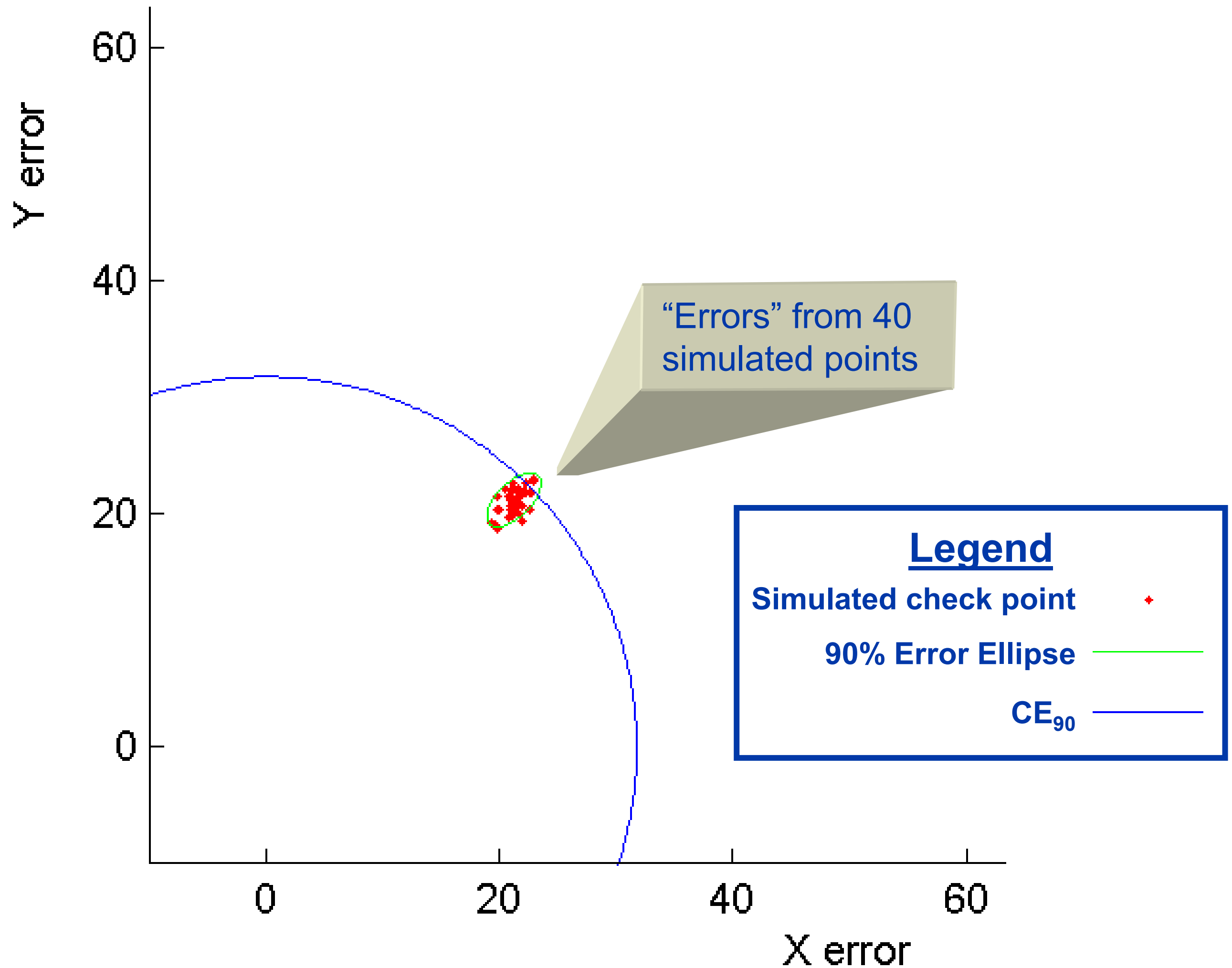
# Example Trial



$$\text{Bias Direction} = 45^\circ$$

$$\frac{\sigma_{\min}}{\sigma_{\max}} = 0.5$$

$$\frac{\mu_H}{\sigma_C} = 30$$

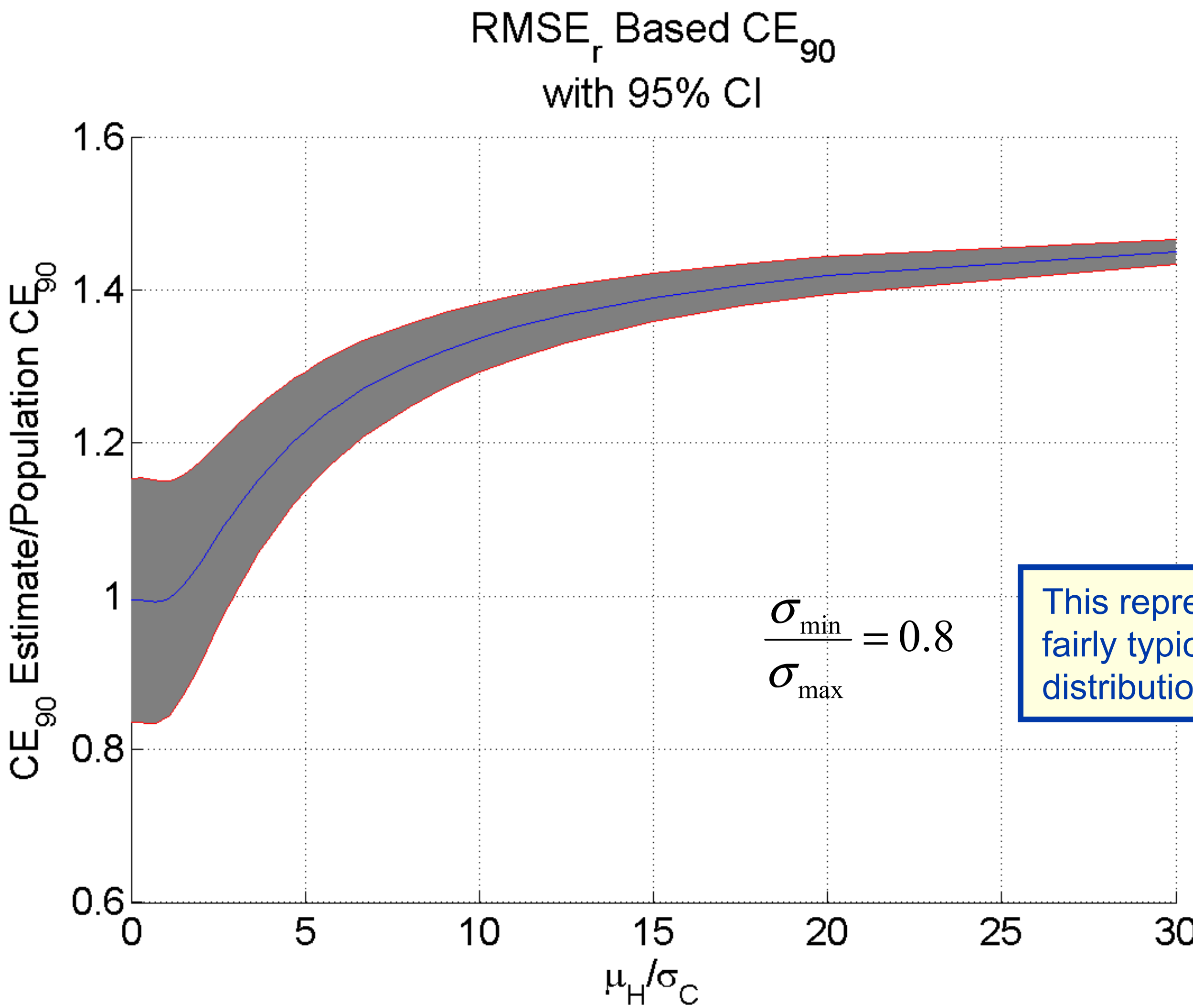


# NSSDA $\text{RMSE}_r$ Based



$$CE_{90} = 1.5175 \cdot \text{RMSE}_r$$

# NSSDA $RMSE_r$ Based Confidence Interval




$$\frac{\sigma_{\min}}{\sigma_{\max}} = 0.8$$

This represents a fairly typical error distribution shape

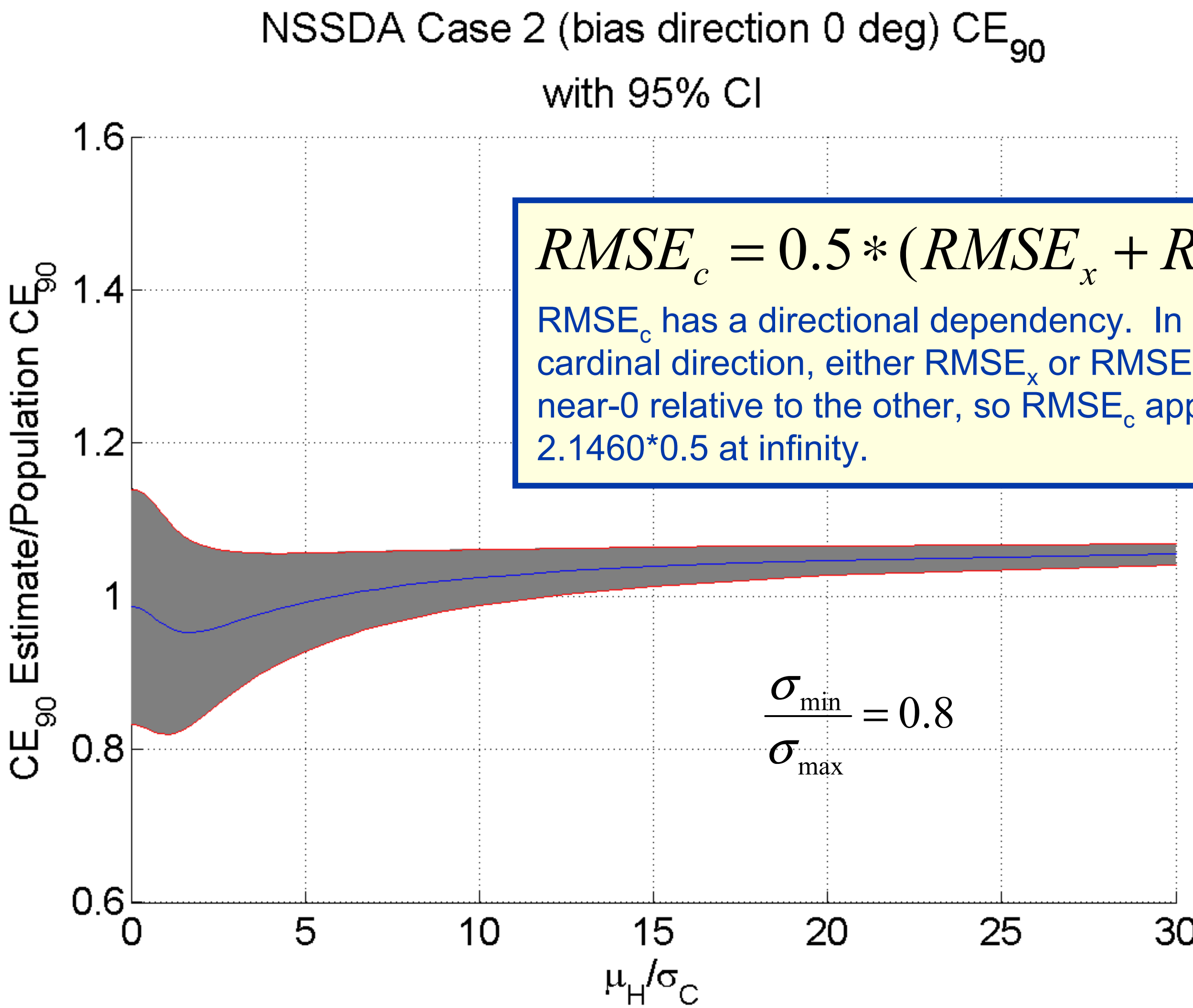
# NSSDA Case 2



$$CE_{90} = 2.1460 \cdot \mathrm{RMSE}_{\mathrm{c}}$$

# NSSDA Case 2 Confidence Interval (cardinal direction)





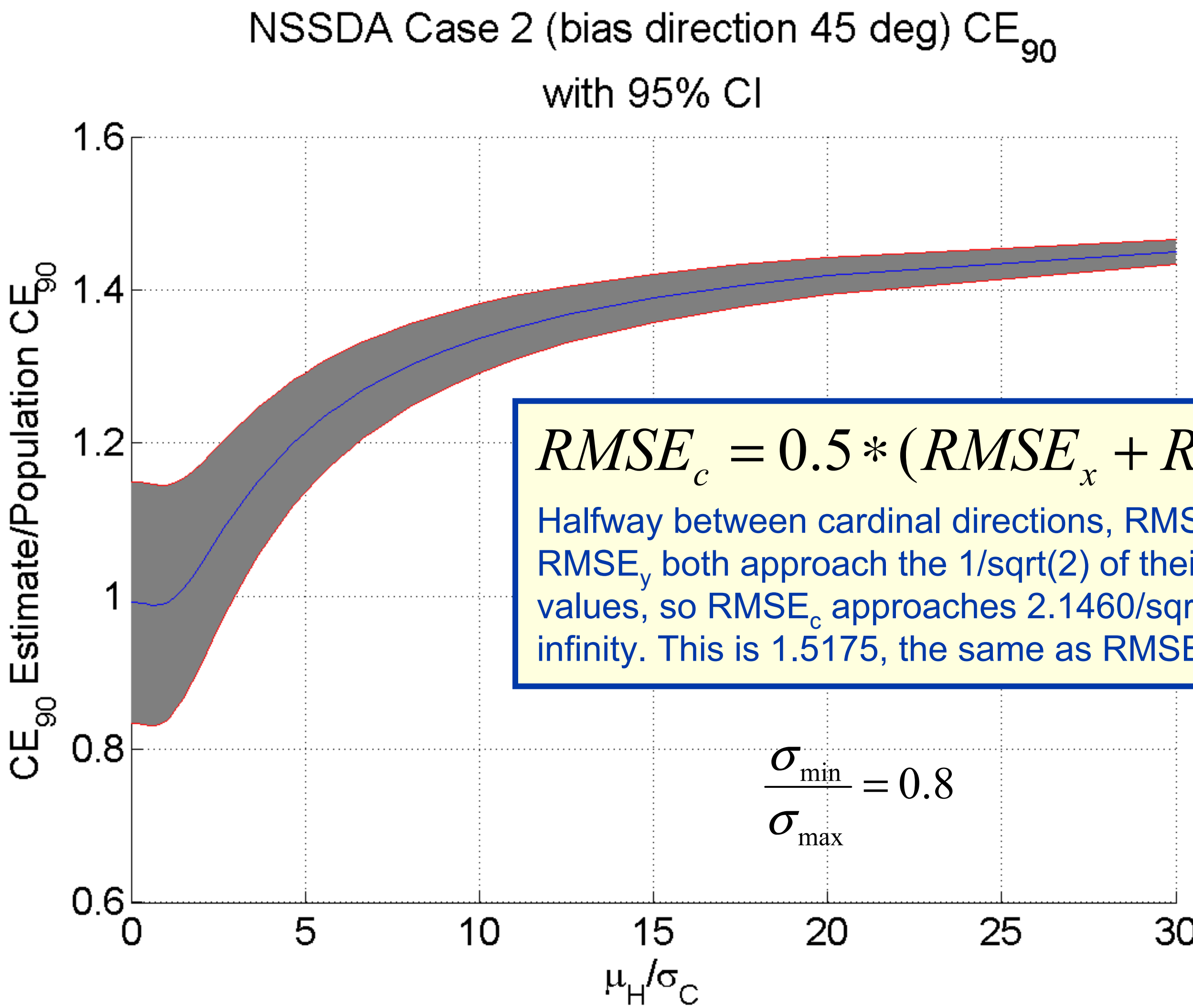

$$RMSE_c = 0.5 * (RMSE_x + RMSE_y)$$

$RMSE_c$ has a directional dependency. In any cardinal direction, either $RMSE_x$ or $RMSE_y$ falls to near-0 relative to the other, so $RMSE_c$ approaches 2.1460*0.5 at infinity.

$$\frac{\sigma_{\min}}{\sigma_{\max}} = 0.8$$

# NSSDA Case 2 Confidence Interval (45° off axis)






$$RMSE_c = 0.5 * (RMSE_x + RMSE_y)$$

Halfway between cardinal directions, $RMSE_x$ and $RMSE_y$ both approach the 1/sqrt(2) of their cardinal values, so $RMSE_c$ approaches 2.1460/sqrt(2) ) at infinity. This is 1.5175, the same as $RMSE_r$.

# Sum of Squares



$$CE_{90} = \sqrt{\left(2.1460 \cdot \sigma_{\mathrm{C}}\right)^2 + {\mu_H}^2}$$

# Sum of Squares Confidence Interval

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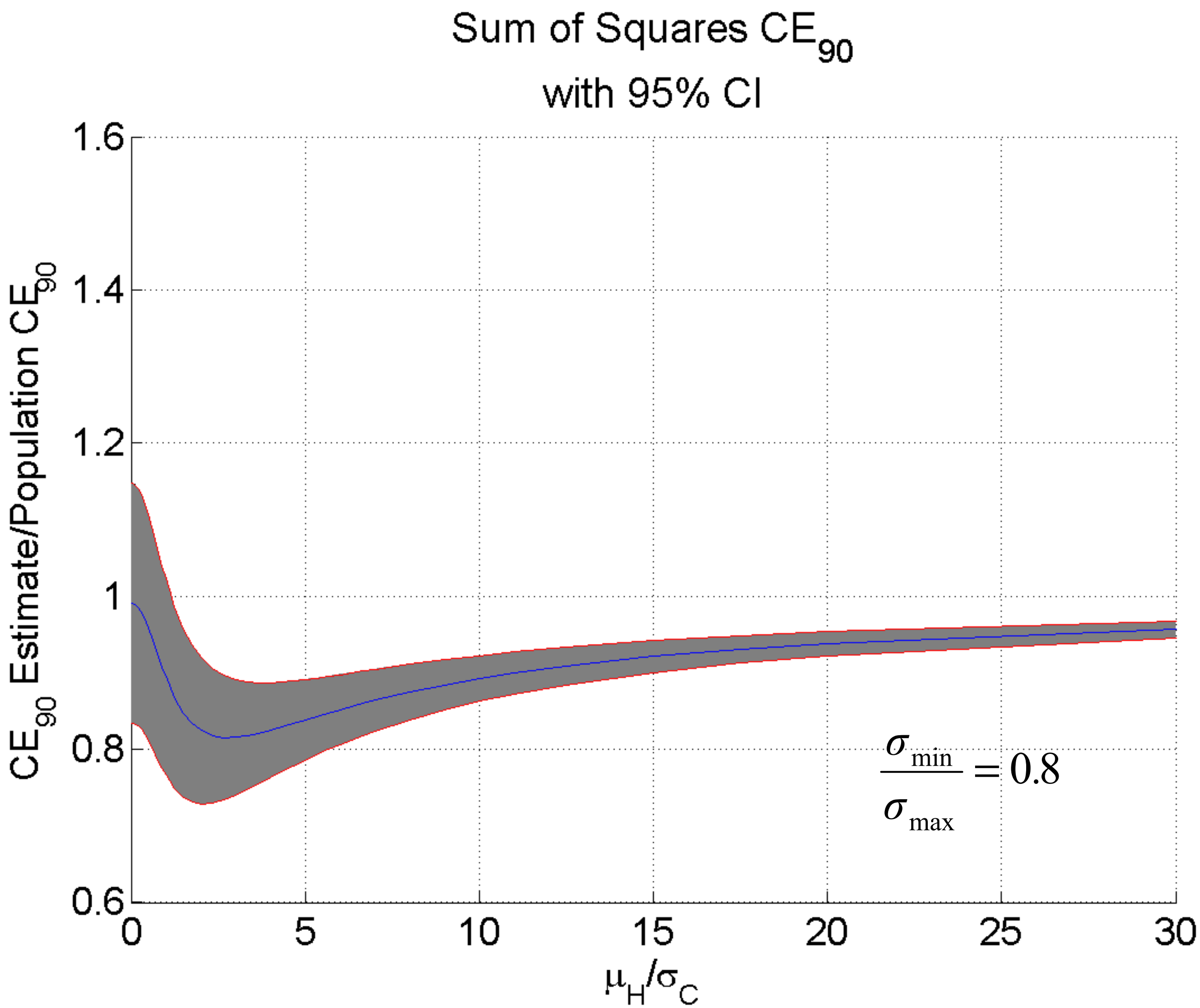
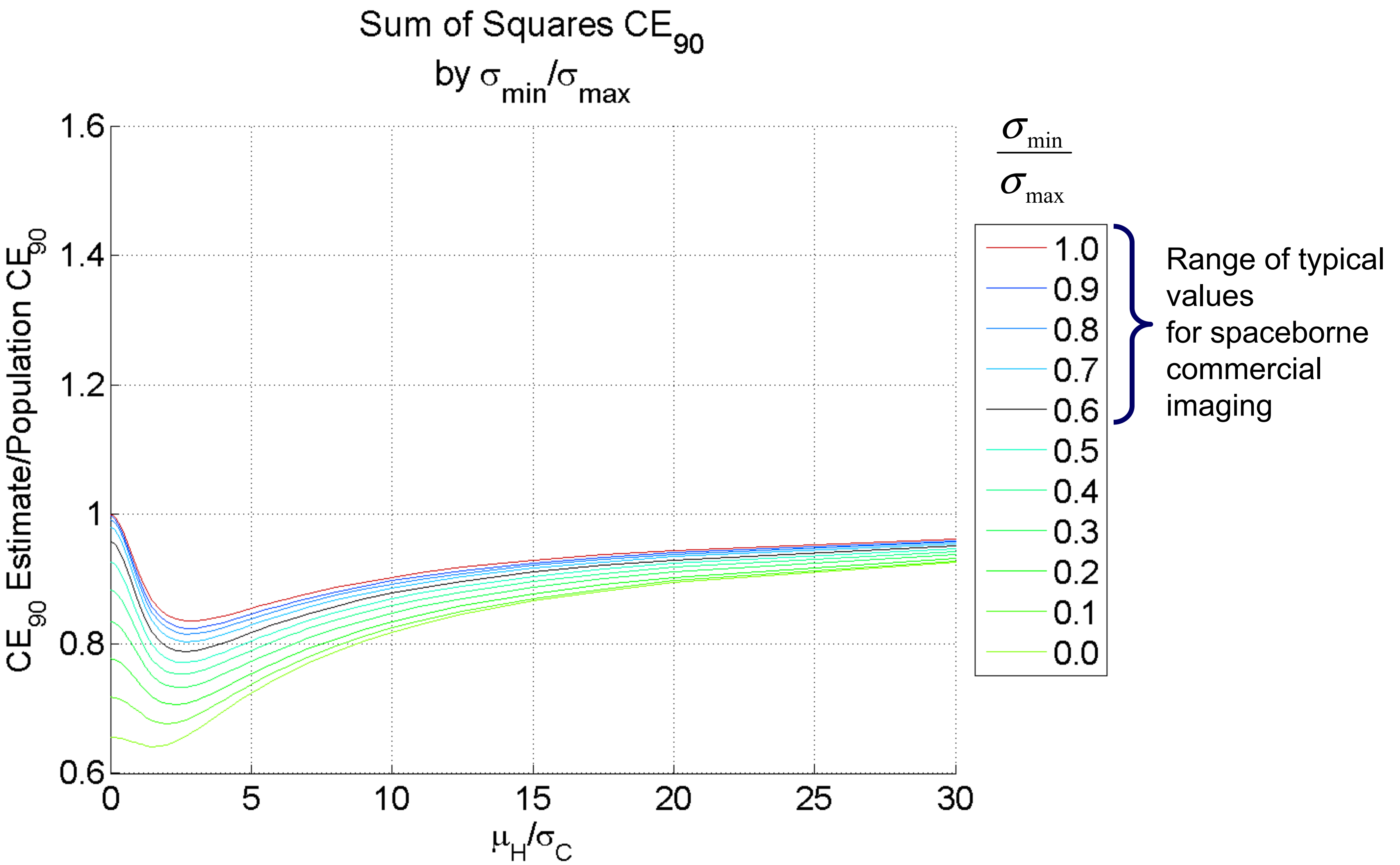


# Sum of Squares Results by Error Distribution Shape





# Ager Approach



- When $\mu_H/\sigma_C \leq 0.1$

$$CE_{90} = 2.1460\sigma_C$$

- When $0.1 < \mu_H/\sigma_C \leq 3$

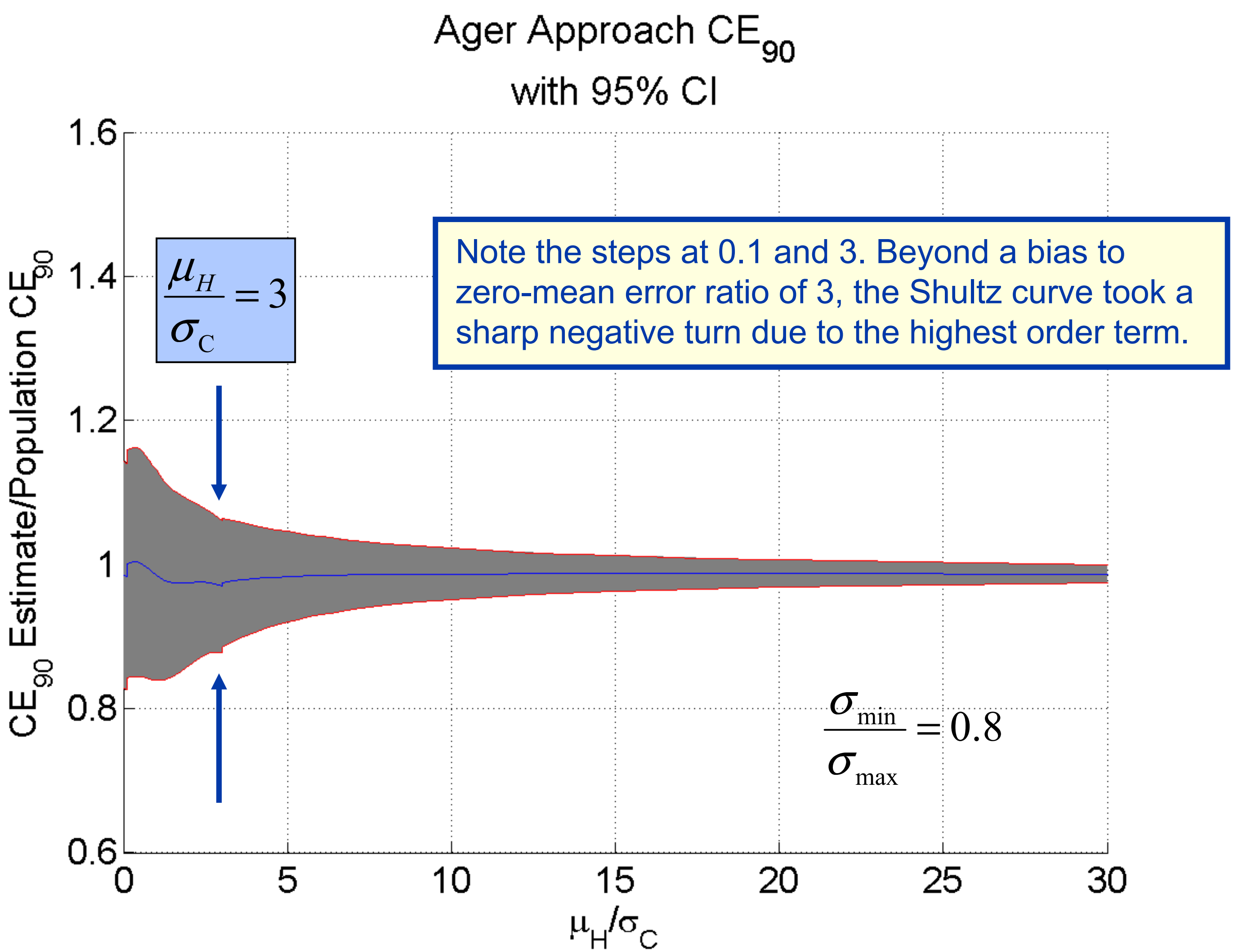
$$CE_{90} = 2.1272\sigma_C + 0.1674\mu_H + 0.3623\frac{\mu_H{}^2}{\sigma_C} - 0.055\frac{\mu_H{}^3}{\sigma_C{}^2}$$

- When $\mu_H/\sigma_C > 3$

$$CE_{90} = 0.986\mu_H + 1.4548\sigma_C$$

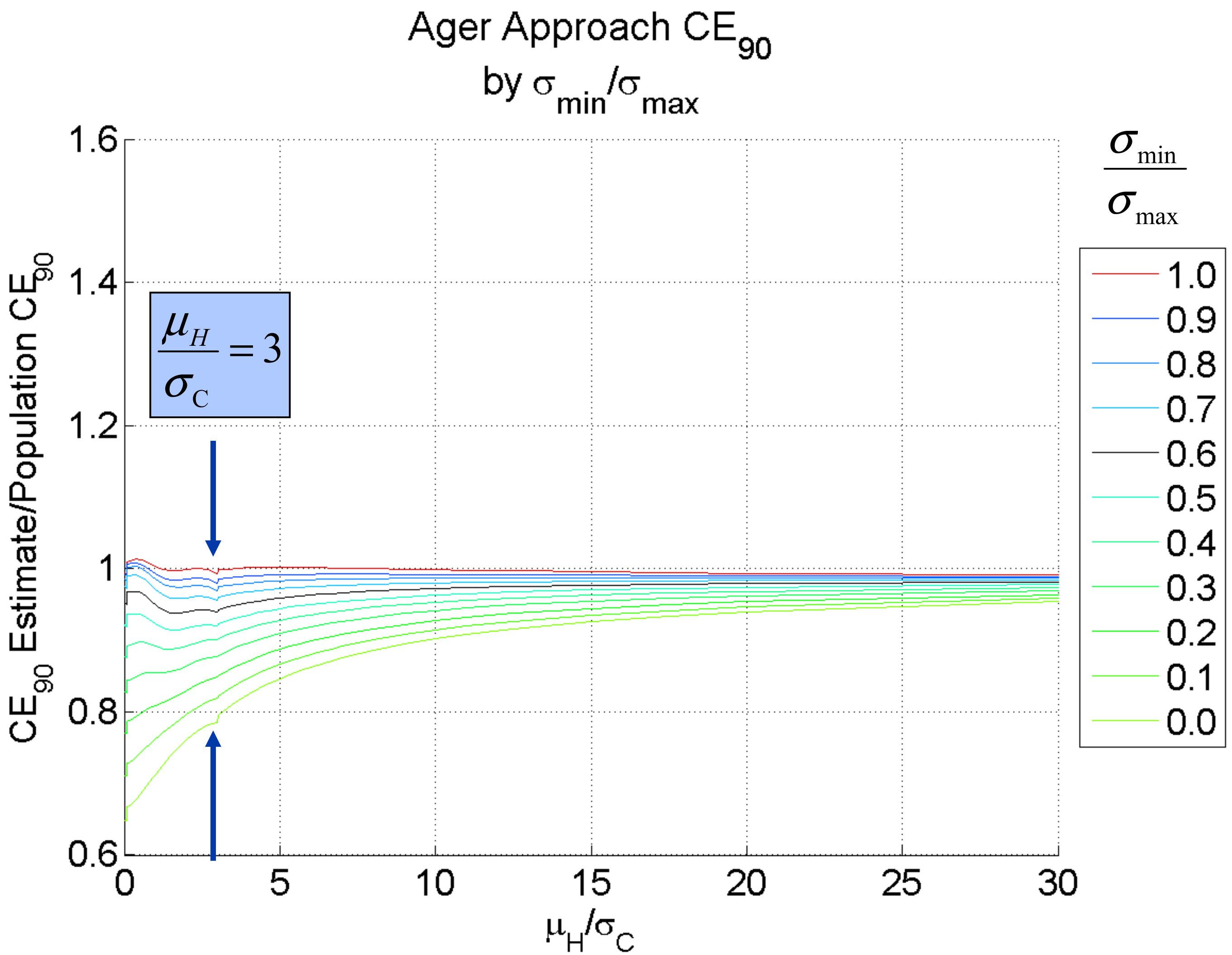
# Ager Approach Confidence Interval





# Ager Approach Results by Error Distribution Shape





# “Empirical” Approach



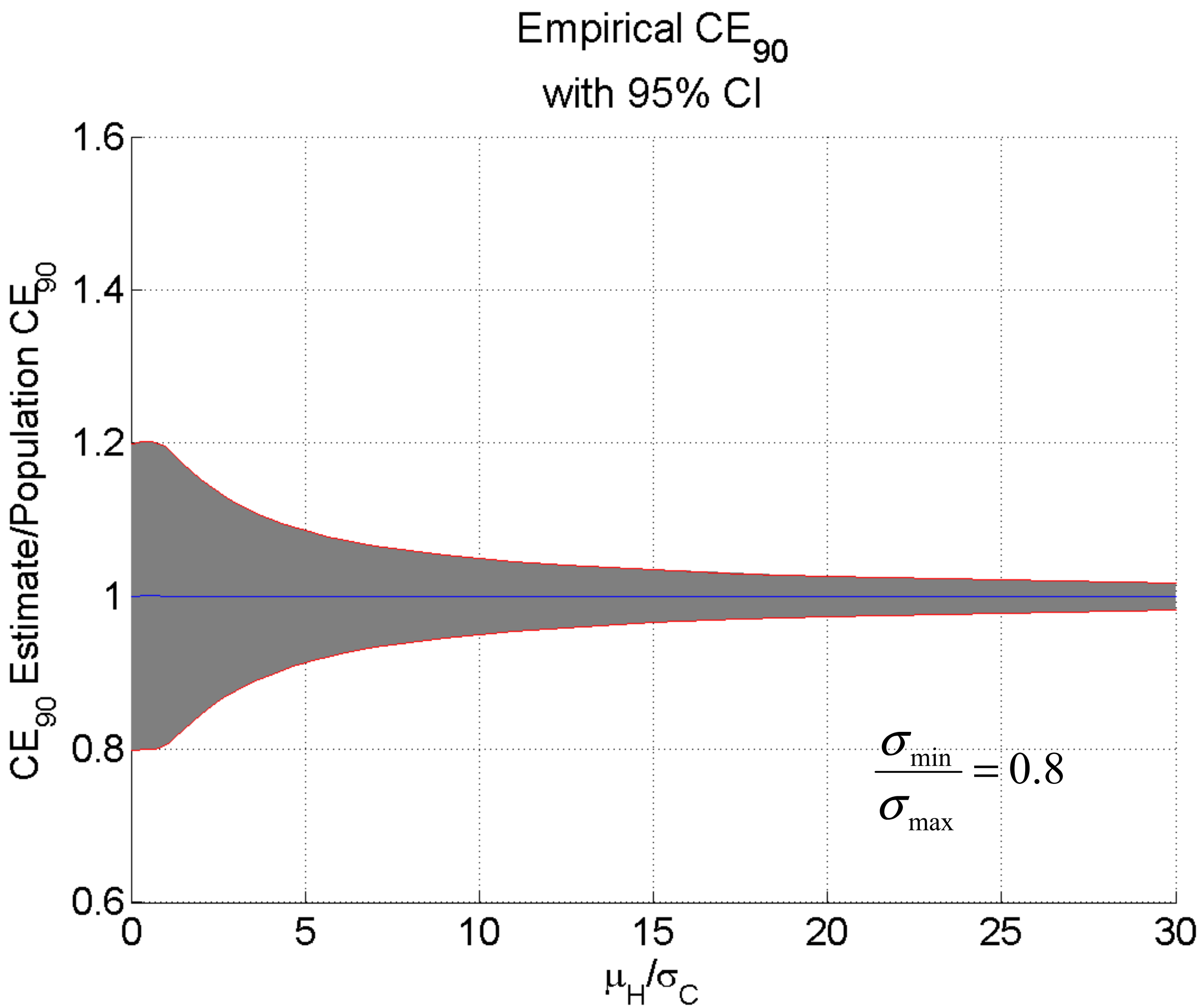
- Given radial error magnitude calculated by

$$\Delta R_i = \sqrt{\Delta X_i^{\;2} + \Delta Y_i^{\;2}}$$

$$CE_{90} = 90^{\text{th}} \text{ percentile of } \Delta R$$

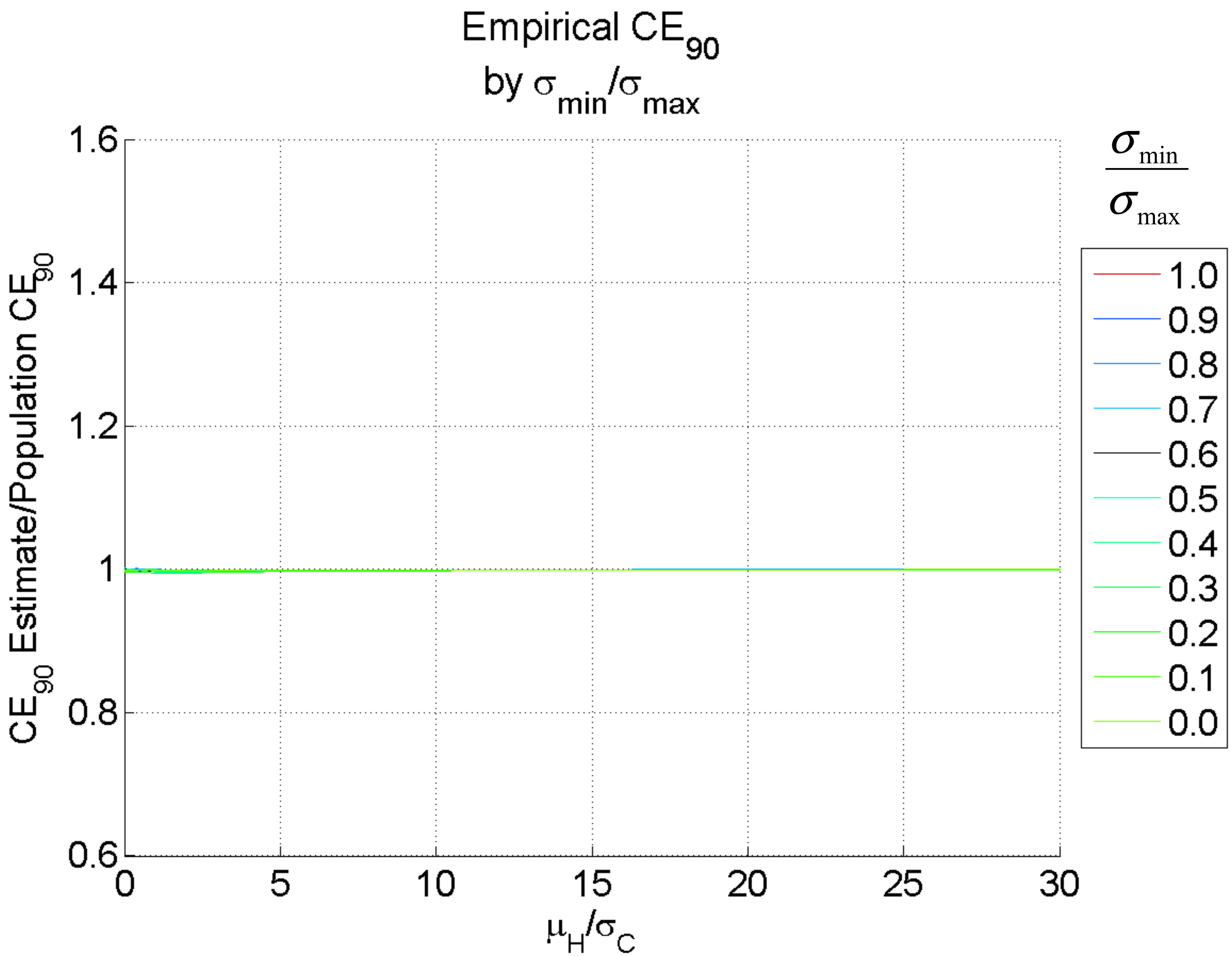
# "Empirical" Approach Confidence Interval





# "Empirical" Approach Results by Error Distribution Shape

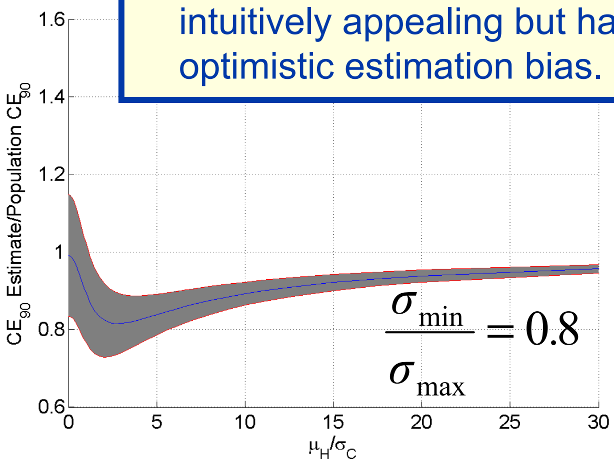




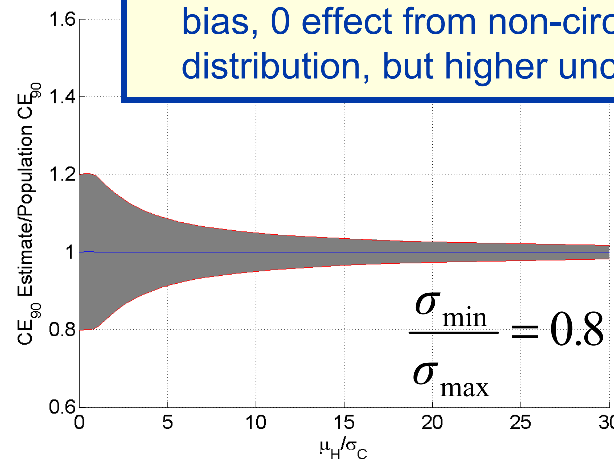
# Side-by-Side Summary



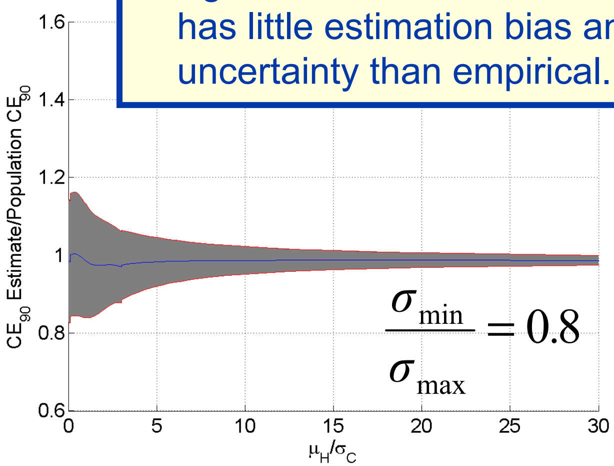
- Sum of squares is simple and intuitively appealing but has optimistic estimation bias.

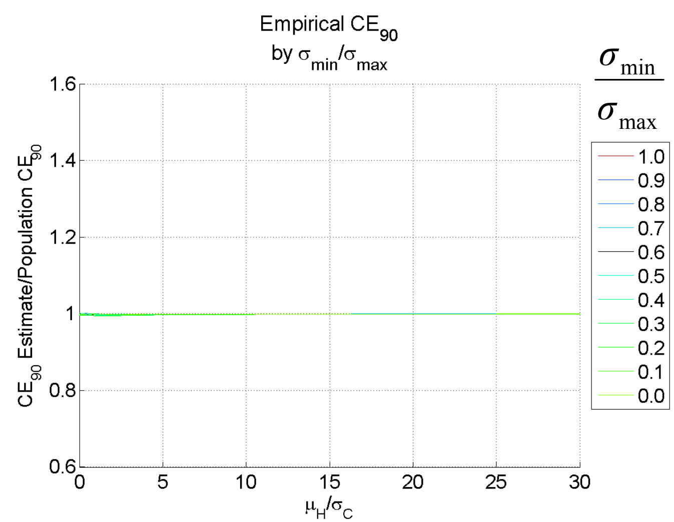



- Empirical method has 0 estimation bias, 0 effect from non-circular distribution, but higher uncertainty.




- Ager modification to Shultz method has little estimation bias and less uncertainty than empirical.



# Conclusions and Recommendations



- RMSE based methods distort circular error estimates (up to 50% overestimation).
- The empirical approach is the only statistically unbiased estimator offered.
- Ager modification to Shultz approach is nearly unbiased, but cumbersome.
- All methods hover around 20% uncertainty (@ 95% confidence) for low geopositional bias error estimates. *This requires careful consideration in assessment of higher accuracy products.*