# SSC Geometric Accuracy Assessment of High Spatial Resolution Commercial Imagery

## 17 A-Order GPS Monuments Onsite

## 45 GPS-Surveyed Geodetic Targets < 3 cm RMSE

Forty-five, 2.44-meter outer radius, 0.6-meter red center radius geodetic targets evenly spaced throughout site

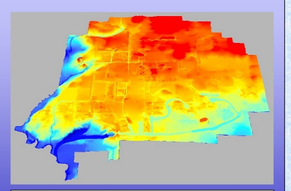
## 150 GPS-Surveyed Painted Manhole Covers < 3 cm RMSE

150 painted manhole covers, ranging in size from 0.86 meters to 2.533 meters, spaced throughout the site.

## 76 GPS-Surveyed Road Intersection Points

## SSC Terrain Effects

- **Terrain at SSC is generally very flat**
  - –Statistics in meters referenced to the WGS84 ellipsoid
    - Average Height: -19.48 m
    - Max Height: -17.40 m
    - Min Height: -24.95 m
    - Standard Deviation in Height: 1.69 m



Color DEM of SSC

## RTK GPS Survey Techniques

Trimble 4000SSi and Trimble 5700 GPS units were used with Pacific Crest radio modems to create a Real-Time-Kinematic (RTK) surveying system. A base station GPS unit was set up on one of many A-order monuments at SSC (many different A-order monuments were used to complete this project). Another GPS unit was used as a rover to survey each target location.

## Fundamentals of $CE_{90}$

A standard metric often used for horizontal accuracy in map or image products is circular error at the 90% confidence level ($CE_{90}$). The National Map Accuracy Standard (NMAS) established this measure in the U.S. geospatial community. NMAS (U.S. Bureau of the Budget, 1947) set the criterion for mapping products that 90% of well-defined points tested must fall within a certain radial distance. According to Greenwalt and Shultz (1968), if X and Y errors are part of a bivariate normal distribution and the X and Y errors are equal, independent, and zero-mean, then

$$CE_{90} = 2.1460 \cdot RMSE_x \quad (1)$$

where $RMSE_x$ is the root mean square summary of X error in tested points defined as follows:

$$RMSE_x = \sqrt{\frac{\sum (X_{image} - X_{control})^2}{n}} \quad (2)$$

$RMSE_y$ is computed in the same fashion.

Frequently, $RMSE_x$ and $RMSE_y$ are summarized by their radial equivalent, the magnitude of their vector sum, $RMSE_r$

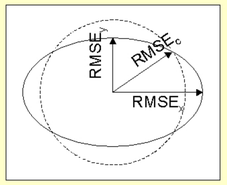
$$RMSE_r = \sqrt{(RMSE_x)^2 + (RMSE_y)^2} \quad (3)$$

The National Standard for Spatial Data Accuracy (NSSDA) states that $CE_{90}$ in terms of $RSME_r$ is given as follows:

$$CE_{90} = 2.1460 \cdot RMSE_r / 1.4142 \quad (4)$$

$$CE_{90} = 1.5175 \cdot RMSE_r$$

This is easily derived from Equations 1 and 3 given that $RMSE_x = RMSE_y$.




Relationship of $RMSE_x$, $RMSE_y$, and $RMSE_c$

## Table 1

| $\frac{RMSE_{min}}{RMSE_{max}}$ | $\frac{RMSE_c}{RMSE_{max}}$ |
|---|---|
| 1.0000 | 1.0000 |
| 0.8165 | 0.9063 |
| 0.6547 | 0.8197 |
| 0.5000 | 0.7323 |
| 0.3333 | 0.6327 |
| 0.2294 | 0.5727 |
| 0.1005 | 0.5274 |
| 0.0 | 0.5151 |

## Calculating $CE_{90}$ Without Removing a Bias (FGDC Standard)

When X and Y errors are not equal, however, Greenwalt and Shultz (1968) point out that Equation 4 is less accurate for estimating circular error than a linear combination of $RMSE_x$ and $RMSE_y$ given as follows:

$$RMSE_c = 0.5222 \cdot RMSE_{min} + 0.4778 \cdot RMSE_{max} \quad (5)$$

where $RMSE_{min}$ is the minimum value between $RMSE_x$ and $RMSE_y$, and where $RMSE_{max}$ is the maximum value of the pair. This linear combination was shown to be a good estimate of the circular error equivalent value $RMSE_{min}$ to $RMSE_{max}$ ratios between 0.6 and 1.0.

Furthermore, it was shown that a good approximation for $RMSE_c$ is given by

$$RMSE_c \cong 0.5 \cdot (RMSE_{min} + RMSE_{max}) \quad (6)$$

In cases where $RMSE_{min} / RMSE_{max}$ is between 0.6 and 1.0, Equation 6 was used in the U.S. standard for spatial data, by the NSSDA (Federal Geographic Data Committee, 1998). In terms of $RMSE_c$, the estimate for $CE_{90}$ looks the same as Equation 1

$$CE_{90} = 2.1460 \cdot RMSE_c \quad (7)$$

In cases where $RMSE_{min} / RMSE_{max}$ is between 0.6 and 0.2, it becomes necessary to calculate $RMSE_c$ utilizing an interpolated value from statistical data that relates $RMSE_{min} / RMSE_{max}$ to $RMSE_c / RMSE_{max}$ found in Table 1 (Greenwalt and Schultz, 1968).

## Calculating $CE_{90}$ By Removing a Bias

For data georeferenced with data adjustment techniques requiring zero-mean error as one of the conditions for solution, Equations 1, 4, and 7 may give reasonable estimates of $CE_{90}$. If, however, georeferencing is primarily dependent on sensor position and orientation, then any errors in sensor position and orientation may translate into a bias in the coordinate solution. Mikhail and Gracie (1997) related bias and random error to overall error in the following formula:

$$M^2 = \sigma^2 + \beta^2 \quad (8)$$

where $M$ is the root mean square error, $\sigma$ is the random error, and $\beta$ is the bias. The bias is estimated as follows:

$$\beta = \sqrt{\beta_x^2 + \beta_y^2} \quad (9)$$

where $\beta_x$ is the mean X error and $\beta_y$ is the mean Y error. The random error is found by first computing the random error in both the X and Y components. The computation for X is computation is shown here:

$$\sigma_x = \sqrt{\left(\frac{\sum (X_{image} - X_{control} - \beta_x)^2}{n}\right)} \quad (10)$$

With bias removed from the error in the components, a summary measure of random error (circular error, $\sigma_c$, or vector sum, $\sigma_r$) can be computed as in Equations 3, 5 or 6 above by simply replacing $RMSE$ with $\sigma$ as shown in Equations 11, 12, and 13.

$$\sigma_r = \sqrt{(\sigma_x)^2 + (\sigma_y)^2} \quad (11)$$

$$\sigma_c = 0.5222 \cdot \sigma_{min} + 0.4778 \cdot \sigma_{max} \quad (12)$$

$$\sigma_c \cong 0.5 \cdot (\sigma_{min} + \sigma_{max}) \quad (13)$$

Also, in the case of $\sigma_{min}/\sigma_{max} < 0.6$, circular error could be computed by interpolation from Table 1 as discussed above and shown in Equations 14 and 15.

$$\sigma_c = \sigma_{max} \cdot C_{interpolated} \quad (14)$$

$$CE_{90} = \sqrt{(2.1460 \cdot \sigma_c)^2 + \beta^2} \quad (15)$$

## Calculating $CE_{90}$ By Removing a Bias: Empirical Method for Calculating $CE_{90}$

Again, taking the circular error approach, a bias adjusted expression for $CE_{90}$ is derived as follows:

$$CE_{90} = \sqrt{(2.1460 \cdot \sigma)^2 + \beta^2} \quad (16)$$

Equation 10 still assumes that the bias is a constant vector and that the error is bivariate normally distributed. If these assumptions do not hold, a final option is to determine empirically by computing the magnitude of the error vector for each test point:

$$\varepsilon_R = R_{image} - R_{control} = \sqrt{(X_{image} - X_{control})^2 + (Y_{image} - Y_{control})^2} \quad (17)$$

Then $CE_{90}$ is empirically determined as the 90th percentile of $\varepsilon_R$.

## References


Greenwalt, C.R., and M.E. Schultz, 1968. *Principles of Error Theory and Cartographic Applications*, ACIC Technical Report No. 96, Aeronautical Chart and Information Center, U.S. Air Force, St. Louis, Mo., 89 p.

Federal Geographic Data Committee, 1998. *Geospatial Positioning Accuracy Standards Part 3: National Standard for Spatial Data Accuracy*, FGDC-STD-007.3-1998, Federal Geographic Data Committee, Washington D.C., 25 p.

Mikhail, E.M., and G. Gracie, 1997. *Analysis and Adjustment of Survey Measurements*, Van Nostrand and Reinhold Company, New York, N.Y., 368 p.

U.S. Bureau of the Budget, 1947. *United States National Map Accuracy Standards*, U.S. Bureau of the Budget, Washington D.C.



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